

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Simplify $(m^2)^3$

$$m^{2 \times 3}$$

$$m^6$$

(1)

(b) Simplify $x^5 \times x^8$

$$x^{5+8}$$

$$x^{13}$$

(1)

(c) Expand $4p(p^2 + 3p)$

$$4p \times p^2 + 4p \times 3p$$

$$4p^3 + 12p^2$$

$$4p^3 + 12p^2$$

(2)

(Total for Question 1 is 4 marks)

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2 Jonny wants to know how much coffee he will need for 800 people at a meeting.

Each person who drinks coffee will drink 2 cups of coffee.

10.6 g of coffee is needed for each cup of coffee.

Jonny assumes 68% of the people will drink coffee.

(a) Using this assumption, work out the amount of coffee Jonny needs.

Give your answer correct to the nearest gram.

800 people

68% drink coffee

$$0.68 \times 800 = 544 \text{ people}$$

544 people → 2 cups of coffee

$$544 \times 2 = 1088 \text{ cups}$$

$$1088 \times 10.6 \text{ grams}$$

$$= 11532.8 \text{ grams}$$

(nearest gram) →

11533 g
(4)

Jonny's assumption is wrong.

72% of the people will drink coffee.

(b) How does this affect your answer to part (a)?

My answer would be greater as he'd need more coffee.

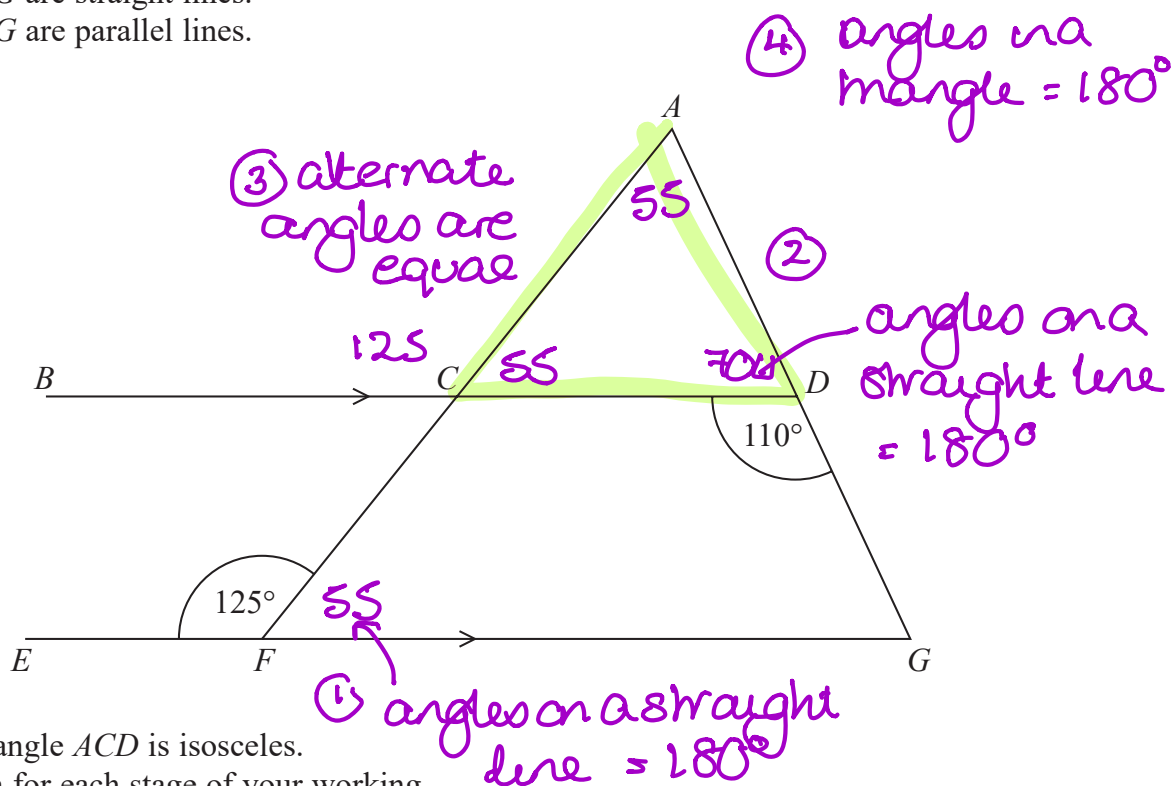
(1)

(Total for Question 2 is 5 marks)



P 7 5 1 5 2 A 0 3 2 4

- 3 ACF and ADG are straight lines.
 BCD and EFG are parallel lines.



Show that triangle ACD is isosceles.
 Give a reason for each stage of your working.

Triangle ACD is isosceles as it has two angles that are the same size.

(Total for Question 3 is 5 marks)



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4 It takes 14 hours for 5 identical pumps to fill a water tank.

inverse proportion

How many hours would it take 4 of these pumps to fill another water tank of the same size?

$$\begin{array}{lcl}
 14 \text{ hours} & = & 5 \text{ pumps} \\
 \times 5 \downarrow & & \downarrow \div 5 \\
 70 \text{ hours} & = & 1 \text{ pump} \\
 \div 4 \downarrow & & \downarrow \times 4 \\
 17.5 \text{ hours} & = & 4 \text{ pumps}
 \end{array}$$

17.5 hours

(Total for Question 4 is 2 marks)



5 A and B are numbers such that

$$A = 2^2 \times 3^4 \times 7$$

$$B = 3^2 \times 7^2$$

(a) Find the highest common factor (HCF) of A and B .

$$\begin{aligned} A &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 \\ B &= \quad \quad 3 \times 3 \quad \quad \quad \times 7 \times 7 \end{aligned}$$

$$\begin{aligned} \text{HCF} &= 3 \times 3 \times 7 \\ &= 9 \times 7 \\ &= 63 \end{aligned}$$

63

(1)

(b) Find the lowest common multiple (LCM) of A and B .

$$\begin{aligned} \text{LCM} &= 63 \times 2 \times 2 \times 3 \times 3 \times 7 \\ &= 15876 \end{aligned}$$

15876

(2)

(Total for Question 5 is 3 marks)



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6 Lava flows from a volcano at a constant rate of $11.9 \text{ m}^3/\text{s}$

How many days does it take for 67205600 m^3 of lava to flow from the volcano?
Give your answer correct to the nearest day.

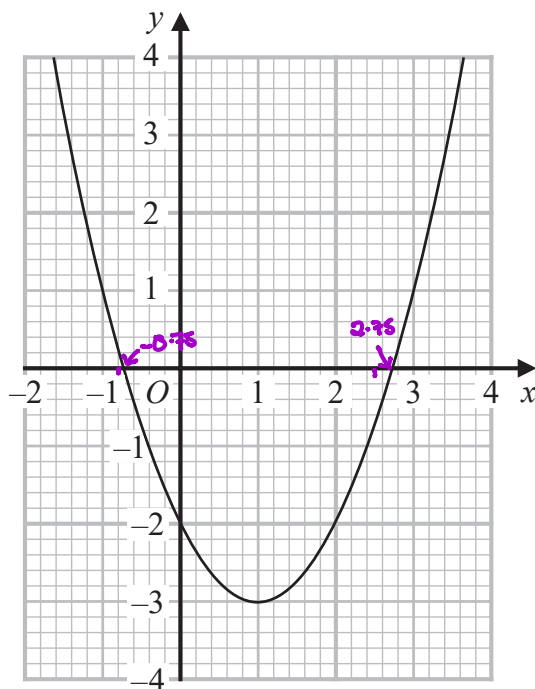
$$\begin{aligned} 67205600 \div 11.9 &= 5,647,529.412 \text{ seconds} \\ &\div 60 \text{ minutes} \\ &\div 60 \text{ hours} \\ &\div 24 \text{ days} \\ &= 65.36492 \dots \\ &65 \text{ nearest day} \end{aligned}$$

..... 65 days

(Total for Question 6 is 3 marks)



7 Here is the graph of $y = x^2 - 2x - 2$



(a) Write down the coordinates of the turning point on the graph of $y = x^2 - 2x - 2$

(1, -3)
(1)

(b) Write down an estimate for one of the roots of $x^2 - 2x - 2 = 0$

$x = -0.75$ $x = 2.75$
range (-0.8 to -0.6, 2.6 to 2.8)
(1)

(Total for Question 7 is 2 marks)



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8 A solid cuboid is made of metal.

The metal has a density of 9 g/cm^3
The volume of the cuboid is 72 cm^3



Work out the mass of the cuboid.

$$\begin{aligned} \text{mass} &= 9 \times 72 \\ &= 648 \end{aligned}$$

..... 648

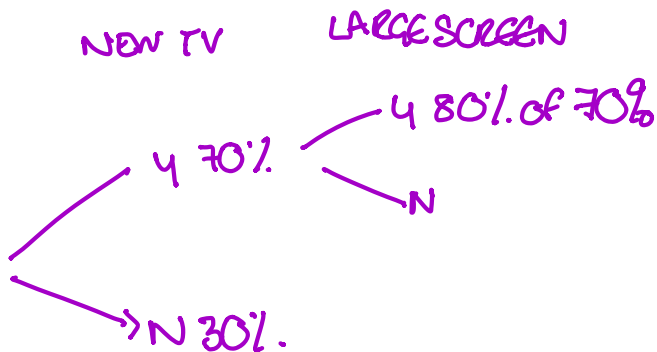
(Total for Question 8 is 2 marks)

9 Some people were asked if they wanted a new television.

70% of the people said yes.

80% of the people who said yes wanted a television with a large screen.

What percentage of the people asked said they wanted a television with a large screen?



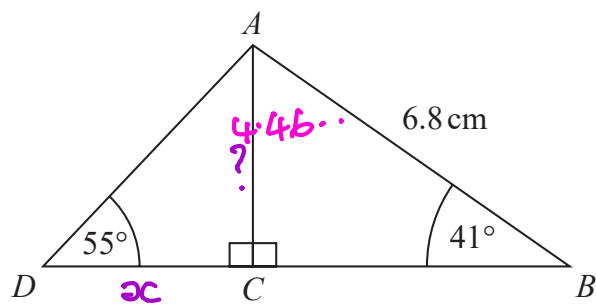
$$0.8 \times 70 = 56$$

..... 56

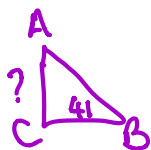
(Total for Question 9 is 2 marks)



- 10 ABD is a triangle.
 C is a point on BD .

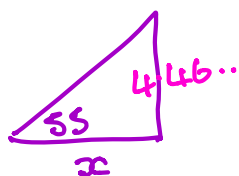


Work out the length of DC .
 Give your answer correct to 1 decimal place.



$$\sin 41 = \frac{AC}{6.8}$$

$$AC = 6.8 \times \sin 41 \\ = 4.46120 \dots$$



$$\tan 55 = \frac{4.46 \dots}{x}$$

$$x = \frac{4.46 \dots}{\tan 55} \\ = 3.1237 \dots \\ = 3.1 \text{ (1dp)}$$

..... 3.1 cm

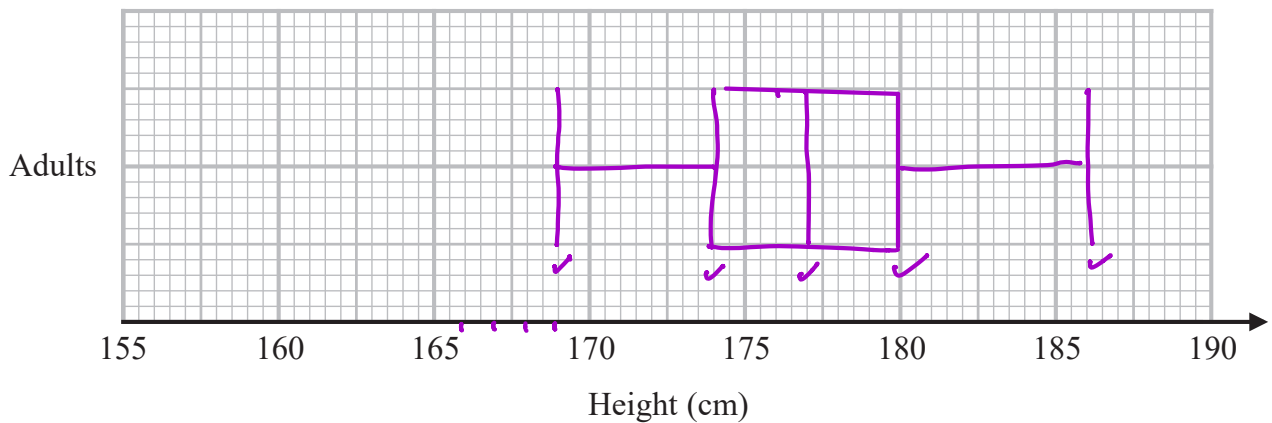
(Total for Question 10 is 3 marks)



11 The table shows some information about the heights of a group of adults.

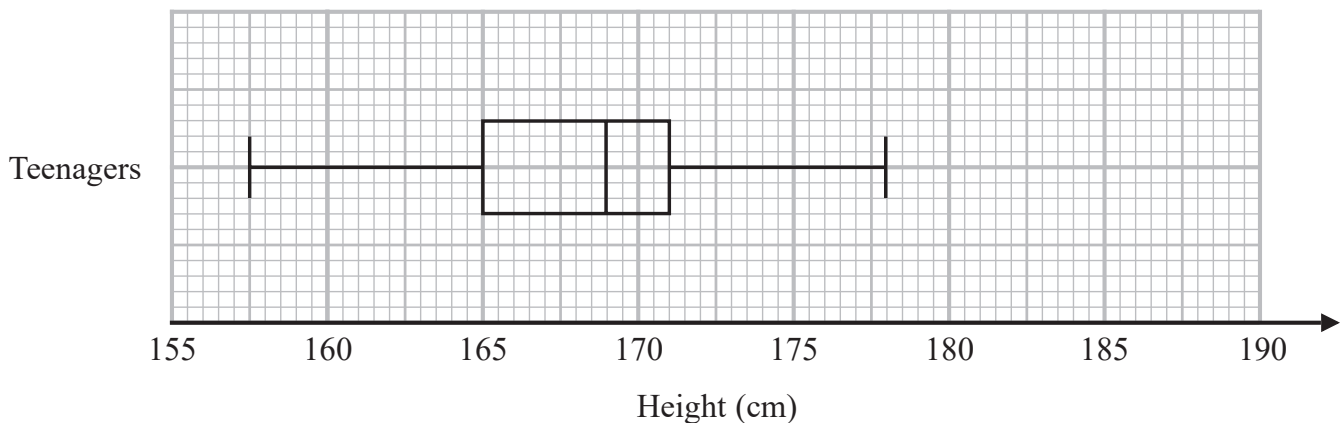
least height	169 cm
greatest height	186 cm
median	177 cm
lower quartile	174 cm
upper quartile	180 cm

(a) On the grid, draw a box plot for the information in the table.



(3)

The box plot below shows the distribution of the heights of a group of teenagers.



(b) Compare the distribution of the heights of the adults with the distribution of the heights of the teenagers. $IQR_{adults} = 180 - 174 = 6$ $IQR_{teenagers} = 171 - 165 = 6$

The median for the adults is greater than the teenagers
so on average the adults are taller.
The IQR of both groups is the same so they have a
similar spread of heights.

(2)

(Total for Question 11 is 5 marks)



- 12 Show that $(x-1)(x+3)(x-5)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$$x^2 + 3x - x - 3 = x^2 + 2x - 3$$

$$(x^2 + 2x - 3)(x - 5)$$

$$= x^3 - 5x^2 + 2x^2 - 10x - 3x + 15$$

$$= x^3 - 3x^2 - 13x + 15$$

where $a = 1$

$b = 3$

$c = -13$

$d = 15$

(Total for Question 12 is 3 marks)

- 13 An expression for the n th term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

$$\frac{n(n+1)}{2} + \frac{(n+1)(n+1+1)}{2}$$

$$= \frac{n^2 + n}{2} + \frac{(n+1)(n+2)}{2}$$

$$= \frac{n^2 + n + n^2 + 3n + 2}{2} = \frac{2n^2 + 4n + 2}{2}$$

$$= n^2 + 2n + 1 = (n+1)(n+1) = (n+1)^2$$

which is a square number.

(Total for Question 13 is 3 marks)

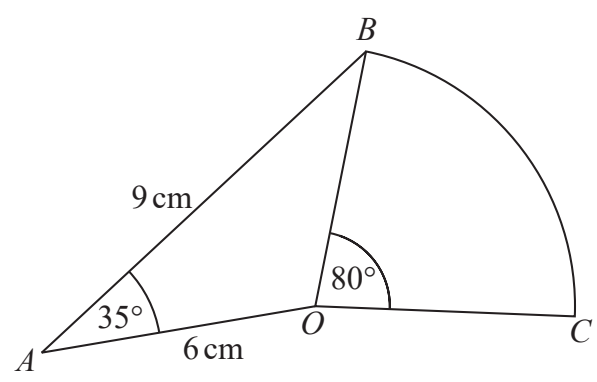


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14 OAB is a triangle.
 OBC is a sector of a circle, centre O .



Calculate the area of OBC .
Give your answer correct to 3 significant figures.

$$BO^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 35$$

$$= 28.53 \dots$$

$$BO = \sqrt{28.53 \dots}$$

$$= 5.341 \dots$$

$$\text{area } OBC = \frac{80}{360} \times \pi \times 5.341 \dots^2$$

$$= 19.9187 \dots$$

$$3\text{s.f.} = 19.9$$

.....19.9..... cm^2

(Total for Question 14 is 4 marks)



15 (a) Factorise $a^2 - b^2$

$$(a-b)(a+b)$$

$$\frac{(a-b)(a+b)}{(1)}$$

(b) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

$$a^2 - b^2$$

$$\text{so } (2^{20})^2 - 1^2$$

$$= (2^{20} - 1)(2^{20} + 1)$$

$$2^{20} = \text{even} \quad 2^{20} - 1 = \text{odd}$$

so $2^{20} + 1$ is next consecutive odd number.

(2)

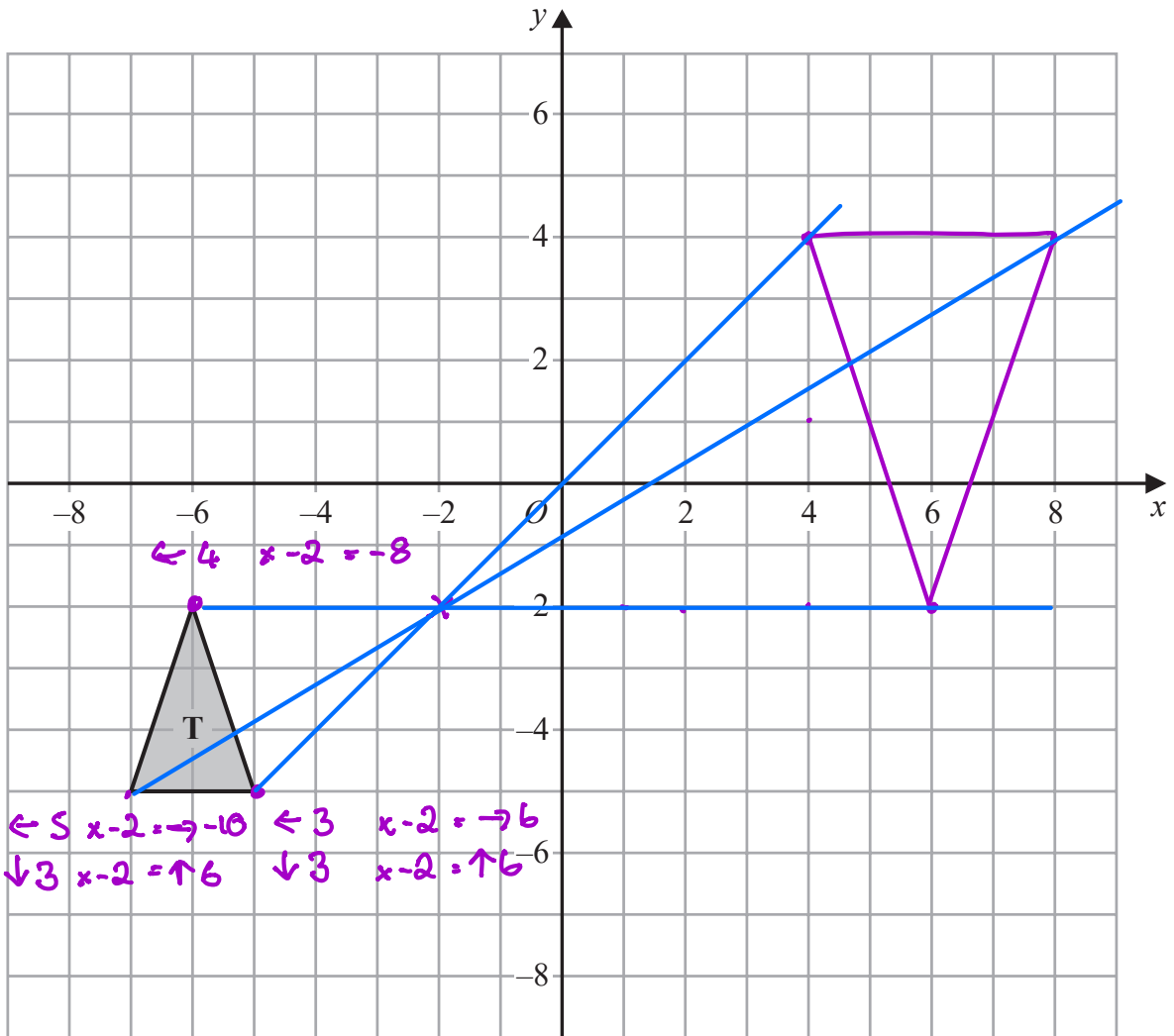
(Total for Question 15 is 3 marks)

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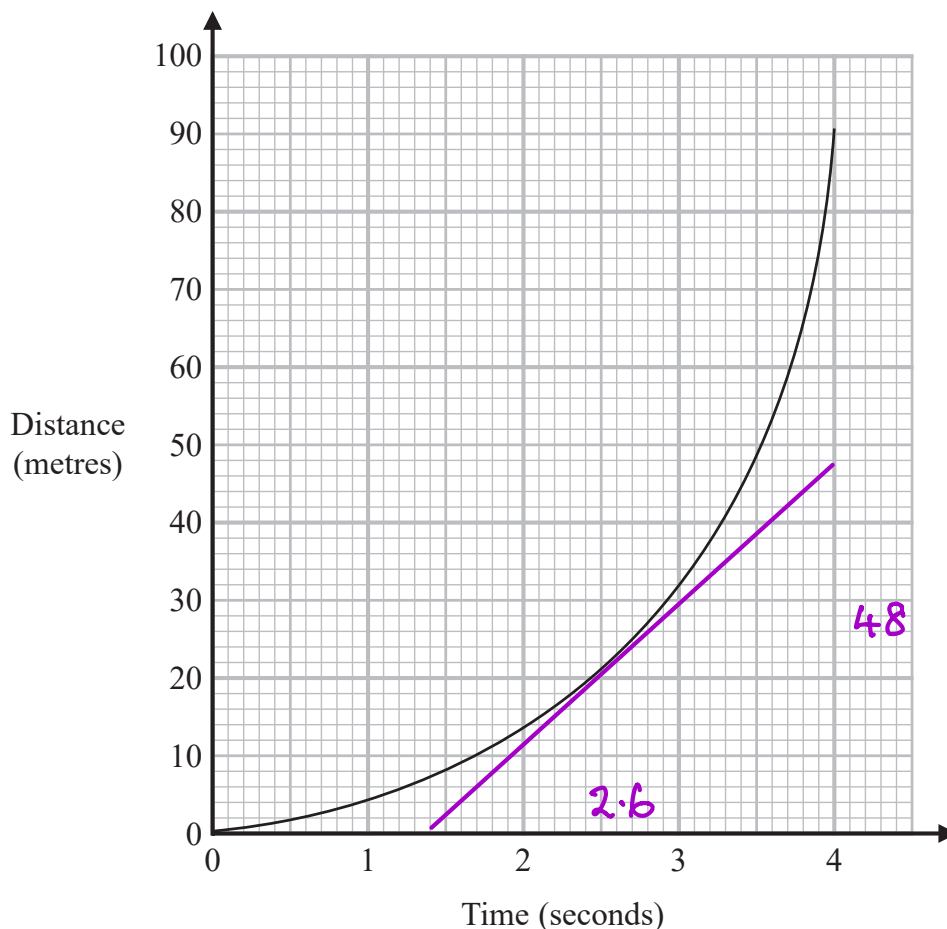


On the grid, enlarge triangle **T** by scale factor -2 with centre of enlargement $(-2, -2)$

(Total for Question 16 is 2 marks)



17 Here is a distance-time graph.



- (a) Find an estimate of the gradient of the graph at time 2.5 seconds. You must show how you get your answer.

$$\text{gradient} = \frac{48}{2.6}$$

$$= 18.46 \dots$$

(This will depend on your tangent line... range of final answers allowed = 18 to 22)

18.5

(3)

- (b) What does the gradient of the graph represent?

Speed (or velocity)

(1)

(Total for Question 17 is 4 marks)

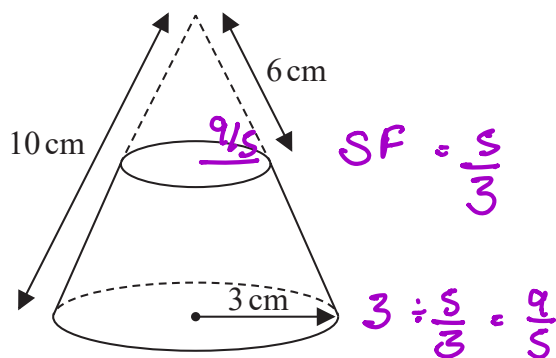


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- 18 A solid frustum is made by removing a small cone from a large cone as shown in the diagram.



Curved surface area of cone = $\pi r l$

The slant height of the small cone is 6 cm.
 The slant height of the large cone is 10 cm.
 The radius of the base of the large cone is 3 cm.

Calculate the total surface area of the frustum.
 Give your answer correct to 3 significant figures.

$$\text{Base} = \pi \times 3^2 = 9\pi \quad \text{Top} = \pi \times \left(\frac{9}{5}\right)^2 = \frac{81}{25}\pi$$

$$\begin{aligned} \text{Curved area} &= \pi \times 3 \times 10 - \pi \times \frac{9}{5} \times 6 \\ &= 30\pi - \frac{54}{5}\pi \\ &= \frac{96}{5}\pi \end{aligned}$$

$$\begin{aligned} \text{Total} &= \frac{96}{5}\pi + 9\pi + \frac{81}{25}\pi \\ &= \frac{786}{25}\pi \\ &= 98.771\dots \end{aligned}$$

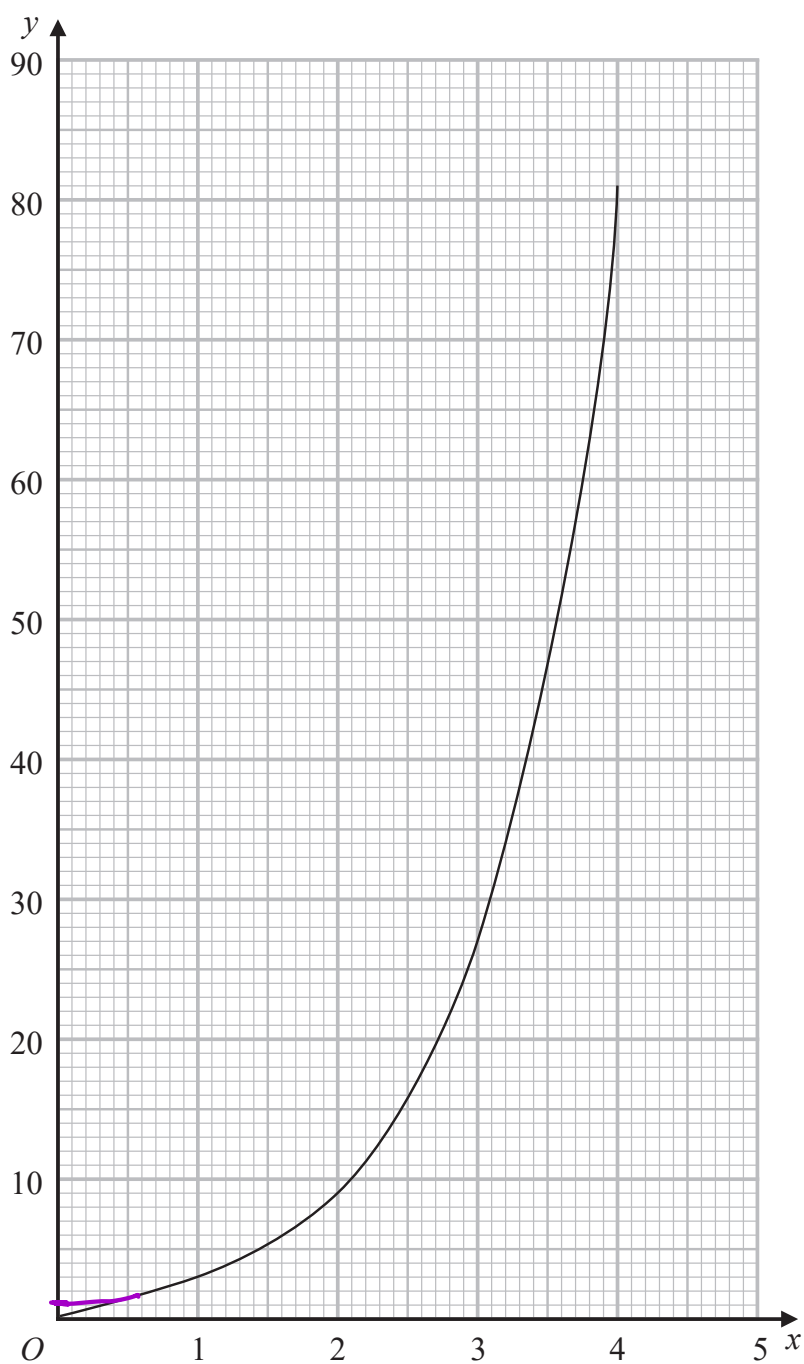
98.8
 (3 s.f.) cm²

(Total for Question 18 is 5 marks)



19 Sana needs to draw the graph of $y = 3^x$ for $0 \leq x \leq 4$

She draws the graph shown on the grid.



Write down one thing Sana has done wrong.

When $x = 0$ $y = 3^0 = 1$ and not zero.
so the line should go through $(0,1)$ not $(0,0)$

(Total for Question 19 is 1 mark)



20 Prove algebraically that $0.1\dot{2}\dot{3}$ can be written as $\frac{61}{495}$

$$\begin{array}{r} 100x = 12.323.. \\ x = 0.1232323... \\ \hline \end{array}$$

$$99x = 12.2$$

$$x = \frac{12.2}{99}$$

$$\frac{12.2}{99} = \frac{122}{990}$$

$$\frac{122}{990} = \frac{61}{495}$$

(Total for Question 20 is 3 marks)

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P 7 5 1 5 2 A 0 1 9 2 4

21 Solve $\frac{1}{x+4} + \frac{3}{2-2x} = 1$

$$2 - 2x + 3(x+4) = (x+4)(2-2x)$$

$$\begin{aligned} 2 - 2x + 3x + 12 &= 2x - 2x^2 + 8 - 8x \\ x + 14 &= -6x + 8 - 2x^2 \end{aligned}$$

$$2x^2 + 7x + 6 = 0$$

$$2 \times 6 = 12$$

$$1, 12,$$

$$2, 6$$

$$\boxed{3, 4}$$

$$2x^2 + 4x + 3x + 6 = 0$$

$$2x(x+2) + 3(x+2) = 0$$

$$(2x+3)(x+2) = 0$$

$$x = -\frac{3}{2} \quad x = -2$$

$$x = -1.5 \quad x = -2$$

(Total for Question 21 is 4 marks)

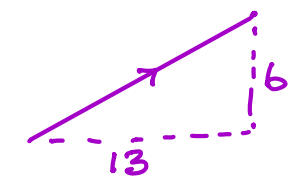


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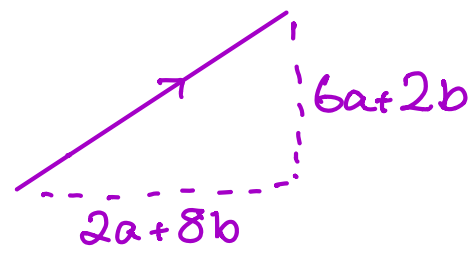
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22 Given that the vector $a\begin{pmatrix} 2 \\ 6 \end{pmatrix} + b\begin{pmatrix} 8 \\ 2 \end{pmatrix}$ is parallel to the vector $\begin{pmatrix} 13 \\ 6 \end{pmatrix}$
 find an expression for b in terms of a .



gradient = $\frac{6}{13}$



① $6a + 2b = b \times 4$

② $2a + 8b = 13$

③ $24a + 8b = 24$

③ - ②

$$\frac{22a}{22a} = 11$$

$$a = 0.5$$

$a = 0.5$ $b = 1.5$
 so $b = 3a$

$b = 3a$

subn ②
 $1 + 8b = 13$

(Total for Question 22 is 3 marks)

$8b = 12$
 $b = \frac{3}{2} = 1.5$



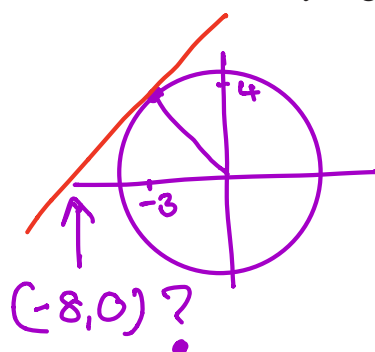
23 A circle has equation $x^2 + y^2 = 25$

The point P with coordinates $(-3, 4)$ lies on the circle.

Alex says that the tangent to the circle at P crosses the x -axis at the point $(-8, 0)$

Is Alex correct?

You must show how you get your answer.



$$x^2 + y^2 = 25 \text{ so radius} = 5$$

$$\begin{aligned} \text{gradient of radius} \\ = -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{gradient of tangent} \\ = \frac{3}{4} \end{aligned}$$

$$\text{equation of tangent } y = \frac{3}{4}x + c$$

$$\begin{array}{l} (-3, 4) \\ x \quad y \end{array}$$

$$4 = \frac{3}{4} \times -3 + c$$

$$c = 4 + \frac{9}{4} = 6.25$$

$$\text{so } y = 0.75x + 6.25$$

$$\text{when } y = 0$$

$$0.75x = -6.25$$

$$x = \frac{-6.25}{0.75} = -8\frac{1}{3}$$

so Alex is not correct.

(Total for Question 23 is 4 marks)



24 There is a total of y counters in a box.

There are x pink counters and 5 blue counters in the box.
The rest of the counters are green.

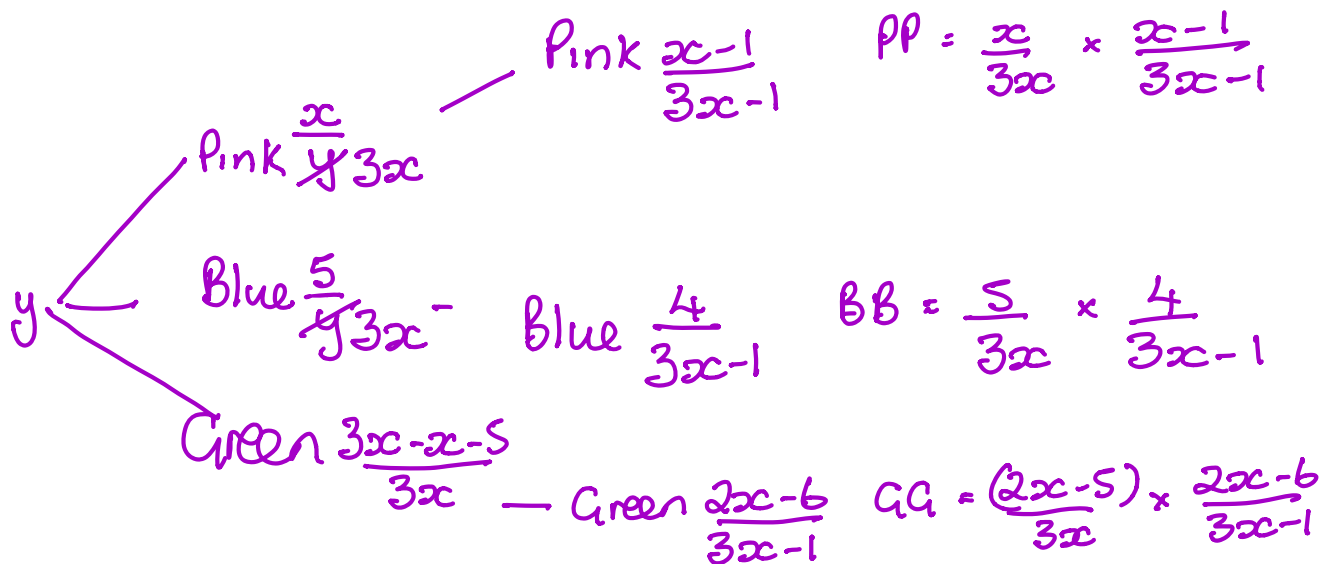
$$x:y = 1:3$$

$$\frac{x}{y} = \frac{1}{3} \quad y = 3x$$

Freda takes at random two counters from the box.

Find, in terms of x , an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form $\frac{ax^2 + bx + c}{dx^2 + ex}$ where a, b, c, d and e are integers.



$$P(\text{same}) = \frac{x^2 - x}{3x(3x-1)} + \frac{20}{3x(3x-1)} + \frac{(2x-5)(2x-6)}{3x(3x-1)}$$

$$= \frac{x^2 - x + 20 + 4x^2 - 12x - 10x + 30}{9x^2 - 3x}$$

$$= \frac{5x^2 - 23x + 50}{9x^2 - 3x}$$

$$\frac{5x^2 - 23x + 50}{9x^2 - 3x}$$

(Total for Question 24 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS

